8. Yu. A. Buevich, B. S. Endler, and I. N. Shchelchkova, "Continuous mechanics of monodispersed suspensions. The rheological equation of state," Preprint No. 85, IPM Akad. Nauk SSSR, Moscow (1977).
9. V. V. Kafarov, A. A. Aleksandrovskii, I. N. Dorokhov, and F. G. Akhmadiev, "Flow of two-phase mixtures in rotary mixers," Inzh.-Fiz. Zh., 32, No. 2, 226-233 (1977).
10. G. A. Korn and T. M. Korn, Manual of Mathematics, McGraw-Hill (1967).
11. B. B. Boiko and N. I. Insarova, "Flow of a viscous Newtonian medium in the cavity of a hyperbolic cylinder and a single cavity hyperboloid of revolution," Inzh.-Fiz. Zh., 29, No. 4, 675-681 (1975).
12. J. Happel and R. Brenner, Hydrodynamics at Low Reynolds Numbers [Russian translation], Mir, Moscow (1976).

MAGNETOGASDYNAMIC MODEL OF CAPILLARY DISCHARGE FROM EVAPORATING WALL

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The article describes the mathematical and physical models of heavy-current capillary discharges. The results of numerical calculation of plasma flow in capillaries are presented.

Capillary discharge from an evaporating wall (CDEW) is widely used in standard light sources ÉV-45 [1] and "Impul's-5" [2] as source of plasma with controlled parameters. This makes it possible to investigate the thermodynamic and optical characteristics of the plasma, and also processes occurring in plasma jets, etc. The experimental study of CDEW is limited on account of its specific features: a relatively cool, optically dense plasma shell adjacent to the walls of the capillary prevents us from obtaining direct information on the parameters prevailing in the hot region of the discharge near the axis. Yet this region may play a decisive part in the overall energy balance and mass balance of the evaporated substance, especially when radiant transfer is the dominant process of energy supply to the wall [3]. Another equally important circumstance stimulating interest in the theoretical investigation of CDEW is the strong nonideality of plasma in capillaries that leads to plasma phase transformation [3]. Also of interest is the study of plasma with higher parameters than in the gasdynamic regime of CDEW, induced and maintained by currents of heavy-current pulse discharge in the gas, the intensity being $\sim 10^5$ A or more [4, 5].

Thus, working out a theoretical model of CDEW and preparing on its basis a program of calculating the dynamics of the phenomenon makes it possible to reveal processes and parameters that are inaccessible to direct experimental observations. This, in turn, makes it possible to influence in a controlled manner the quantitative characteristics of CDEW.

Below we describe the physical and mathematical models of heavy-current capillary discharges when the magnetic eigenfield of the discharge current is of high intensity, and the magnetic pressure is comparable with the gas-kinetic pressure. We also present the results of numerical calculations of radiative and gasdynamic processes occurring in capillary discharges from the initial nonsteady-state phase up to the establishment of steady-state plasma motion.

To describe plasma flow in the channel of a capillary discharge with heavy discharge currents, when magnetic pressure may not be neglected in comparison with the gas-kinetic pressure, we use a system of magnetic and gasdynamic equations supplemented by Maxwell's equations of the electromagnetic field [6]. To construct the model of the flow, we make some estimates. The test of freezing of the magnetic field in the plasma is the magnetic Reynolds number Re_H. The characteristic parameters for the magnetic and gasdynamic (MGD) flow regime of plasma in

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a capillary are $\eta = 5\cdot10^{14}$ sec^{-1}, L = 0.1 cm, W = 10^5 km/sec. Thus, $Re_H < 1$, and therefore in the first approximation we may neglect the term rot [W×H] in the equation of induction $\frac{\partial H}{\partial t} = \mathrm{rot}\,[\mathbf{W}\times\mathbf{H}] - c\,\mathrm{rot}\,\mathbf{E}$. The time of penetration of the magnetic field into the plasma (skin time) for the characteristic parameters in the MGD regime of capillary discharge is $\tau_H = L^2/D_H = 10^{-7}$–$10^{-8}$ sec, and the gasdynamic time is $\tau = 10^{-6}$ sec. Thus the diffusion rate of the magnetic field into the plasma is high compared with the gasdynamic speeds, the magnetic field is able to follow the changes of the gasdynamic parameters of plasma flow, and at any instant it may be considered in steady state.

In magnetic gasdynamics it may be assumed that the dielectric and magnetic permeabilities are equal to unity. Under the conditions of capillary discharge the electric conductivity of the plasma is fairly high, and the characteristic time is ~1 μsec, the displacement current may therefore be neglected in comparison with the conduction current. Hence follows that for winding the magnetic field intensity in the first approximation it suffices to use the equation $\mathrm{rot}\,\mathbf{H} = \frac{4\pi}{c}\mathbf{j}$. In addition to that we assume that the medium is quasineutral, i.e., the total electric charge in any elementary volume is equal to zero.

In the general case conductivity is a tensor characteristic. In magnetized plasma, conductivity and also other transfer coefficients become anisotropic. For nonmagnetized plasma, conductivity may be described by a scalar magnitude that does not depend on the magnetic field intensity. The criterion characterizing magnetization is the product of the cyclotron frequency ω_c of the rotation of a charged particle in the magnetic field and τ_c, the mean pulse transfer time upon interaction of the charged particles. According to estimates in the MGD regime of a capillary discharge, $\omega_c\tau_c < 1$. This condition of nonmagnetization may be infringed only in the near-wall layer where the temperature greatly drops.

Thus, the plasma in the capillary discharge channel may be considered nonmagnetized, the magnetic field is not frozen in, and it penetrates freely into the plasma.

In the present work we take it that the magnetic field has only one nonzero azimuthal component: $H_r = 0$, $H_z = 0$, $H_\varphi = H$; the electric field also has only one nonzero component: $E_r = 0$, $E_\varphi + 0$, $E_z = E = \mathrm{const}$, and the discharge current flows in the direction of the z axis. Thus the volumetric ponderomotoric force of the electric field acts on the plasma only in the radial direction.

Taking into account the estimates and assumptions made above, we may write the system of differential equations describing the magnetic and gasdynamic regime of plasma flow in a capillary discharge in the following manner:

$$\frac{\partial\rho}{\partial t} + \mathrm{div}\,(\rho\mathbf{W}) = 0, \quad \frac{\partial(\rho W_z)}{\partial t} + \mathrm{div}\,(\rho W_z\mathbf{W}) = -\frac{\partial P}{\partial z},$$
$$\frac{\partial(\rho W_r)}{\partial t} + \mathrm{div}\,(\rho W_r\mathbf{W}) = -\frac{\partial P}{\partial r} - \frac{H}{4\pi r}\frac{\partial}{\partial r}(rH),$$
$$j_z = \frac{1}{4\pi r}\frac{\partial}{\partial r}(rH) = \eta\,(E + W_r H), \tag{1}$$
$$\frac{\partial}{\partial t}(\rho\varepsilon) + \mathrm{div}\,(\rho\varepsilon\mathbf{W}) + \mathrm{div}\,(P\mathbf{W}) = \frac{j_z^2}{\eta} - \frac{HW_r}{4\pi r}\frac{\partial}{\partial r}(rH) - \mathrm{div}\,\mathbf{S}.$$

This system of differential equations is written in dimensionless form, and the following characteristic units of measuring magnitudes in the transition to the dimensionless form were used: $\rho_* = 10$ kg/m^3, $W_* = 1$ km/sec, $P_* = 10$ MPa, $r_* = 10^{-2}$ m, $H_* = 7.96\cdot10^5$ A/m, $j_* = 10^5$ A/cm^2, $E_* = 10^{-1}$ V/cm, $\eta_* = 10^6$ S/m, $\varepsilon_* = 1$ MJ/kg, $S_* = 10$ GW/m^2.

The systems of difference equations corresponding to the differential equations (1) is solved by the method of "large particles" [7]. Heating and evaporation of the capillary surface under the effect of an incident radiant energy flux are described by a method explained in [8]. In the case of cylindrical geometry the equation of radiation transport has the following form [9]:

$$\sqrt{1-\mu^2}\left(\cos\varphi\frac{\partial I}{\partial r}-\frac{\sin\varphi}{r}\frac{\partial I}{\partial\varphi}\right)+\mu\frac{\partial I}{\partial z}=\varkappa(I_p-I).$$

We will examine radiation transport along each axis independently of each other. Thus, along the radius we have the equation

$$\sqrt{1-\mu^2}\left(\cos\varphi\frac{dI^r}{dr}-\frac{\sin\varphi}{r}\frac{dI^r}{d\varphi}\right)=\varkappa(I_p-I^r),$$

and along the z axis which coincides with the axis of the capillary:

$$\mu\frac{dI^z}{dz}=\varkappa(I_p-I^z).$$

The condition of distribution of the radiation is taken into account in the forward–backward approximation [10]. If we integrate these equations with respect to an element of the solid angle $d\Omega=d\mu d\varphi$ and introduce the one-sided intensities of radiation $I^r_{1,2}$ and $I^z_{1,2}$ along each axis, we obtain two systems of differential equations. For the radial direction

$$\frac{d(I_1^r-I_2^r)}{dr}+\frac{(I_1^r-I_2^r)}{r}=2\varkappa[2I_p-(I_1^r+I_2^r)],$$

$$\frac{d(I_1^r-I_2^r)}{dr}=-2\varkappa(I_1^r-I_2^r),$$

and for the axial direction

$$\frac{d(I_1^z-I_2^z)}{dz}=2\varkappa[2I_p-(I_1^z-I_2^z)],$$

$$\frac{d(I_1^z+I_2^z)}{dz}=-2\varkappa(I_1^z-I_2^z).$$

If we solve this system by the method of matching [11] with the boundary conditions: $I_2^r=0$ on the surface of the capillary and on the gas–vacuum interface, $I_1^r=I_2^r$ for r = 0, $I_2^z=0$ on the gas–vacuum interface, $I_1^z=I_2^z$ for z = 0, we obtain the radiant fluxes in each cell of the calculation grid.

Flow in the capillary was calculated by the above-explained method. The radius of the capillary r_0 was taken equal to $r_0 = 1.5\cdot10^{-3}$ m, the length $L = 3\cdot10^{-3}$ m. The dimensions of the calculation grid were $\Delta r = 10^{-4}$ m and $\Delta z = 10^{-4}$ m. For evaluating the accuracy of the calculations we carried out test calculations with steps in space one-third and one-half smaller. The relative change of the principal flow parameters did not exceed 10%, which indicates that the accuracy of the calculations is acceptable. In the examined variant of the calculation the plasma was able to flow out freely into the surrounding space. The thermodynamic and optical parameters of the plasma in the capillary and of the vapors of the wall were taken to be equal, and they corresponded to carbon plasma [12].

At the initial instant the plasma pressure in the capillary was $P_H = 7.5$ MPa, density $\rho_H = 1$ kg/m³, and temperature $T_H = 0.91$ eV. In the outer region pressure was $P_H^{ou} = 0.1$ MPa, density $\rho_H^{ou} = 1$ kg/m³, and temperature $T_H^{ou} = 0.026$ eV. The temperature of the capillary surface at the initial instant was also taken to be equal to T = 0.026 eV. The electric conductivity of the carbon plasma η was specified according to the tables of [13].

In the described variant of the calculation the electric field intensity E was assumed to be constant and equal to 0.3162 MV/m. The count was carried out up to the instant corresponding to the steady-state phase of plasma flow. At first the gas is being heated in the capillary, and in consequence of the energy transport by radiation to the pipe wall, a parabolic temperature profile along the radius forms in it. The radiant energy flux, acting on the wall, heats the surface layer of the capillary, and then causes its evaporation. The beginning of this process is associated with the energy absorbed by the surface of the capillary. In the case under examination, evaporation began at the instant t = 0.2 μsec, at the same time the temperature of the surface layer of the capillary was T = 0.5 eV, and the radiant flux of the

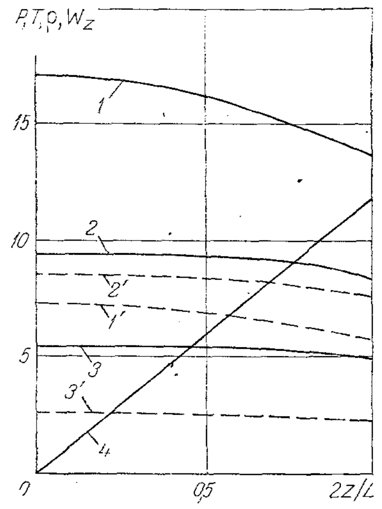

Fig. 1

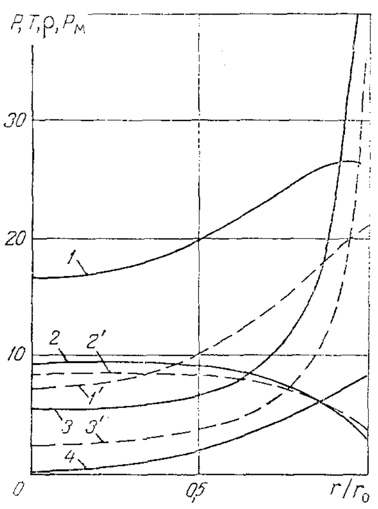

Fig. 2

Fig. 1. Distribution of pressure (1), temperature (2), density (3), of the axial velocity component (4) along the z axis: 1', 2', 3') distribution of the same magnitudes along the z axis not taking the volumetric electromagnetic force into account.

Fig. 2. Distribution of pressure (1), temperature (2), density (3), magnetic pressure (4) in the middle section along the radius: 1', 2', 3') distributions of the same magnitudes without taking the volumetric electromagnetic force into account.

radiation attained its maximum values $\sim$0.22 TW/m^2. Then, in consequence of ablation and the formation of a relatively cool layer of evaporated substance of the wall screening the surface, the flux of radiant energy somewhat diminishes and assumes a steady-state value.

Figure 1 shows the distributions of pressure P (10 MPa), temperature T (eV), density ρ (0.1 kg/m^3), and of the axial velocity component W_z (1 km/sec) along the axis of the capillary. The pressure decreases monotonically toward the edge of the capillary from 0.17 to 0.135 GPa, the axial velocity component increases linearly from zero in the plane of symmetry to 11.8 km/sec at the edge, the temperature decreases from 9.47 to 8.35 eV at the edge. For the sake of comparison, the same figure also shows the distribution of the flow parameters (pressure P', density ρ', and temperature T') along the axis of the capillary for the same variant but without taking into account the effect of the electromagnetic field.

Figure 2 shows the distributions of the flow parameters along the radius for the middle section. Here, P_M is the magnetic pressure. The dashed lines show the pressure, density, and temperature without the effect of the electromagnetic field taken into account.

Figure 3 also shows the distribution of pressure, density, temperature, velocity W_z, magnetic field intensity H (7.96·10^5 A/m), magnetic pressure P_M, of the radial velocity component W_r, of the electric conductivity of the plasma η (10^4 Ω/m) along the radius at the edge of the capillary.

Figure 4 presents the dependences of the density of the radiant flux incident on the surface of the capillary S (10 GW/m^2), of the mass of evaporated substance M_{evp} (2·10^{-8} kg), of the mass of plasma in the capillary m_K (2·10^{-9} kg), and of the energy liberated by the Joulean source Q_{sou} (20 J) on the time. It should be noted that the evaporation rate assumes its steady-state value soon after the onset of evaporation.

Thus, from the presented figures a fairly complete notion of the flow pattern and of the spatial distribution of the parameters in the steady-state phase of plasma motion in a capillary in the MGD regime may be obtained.

It follows from what has been explained that the chosen model of the phenomenon presents its qualitative and quantitative regularities. The model includes the processes of energy intake by ohmic heating of the plasma contained in the capillary, radiant heat exchange of the plasma with the wall, heating and evaporation in consequence of this, taking account of the

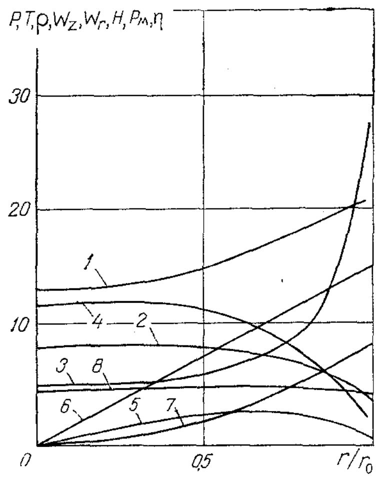

Fig. 3

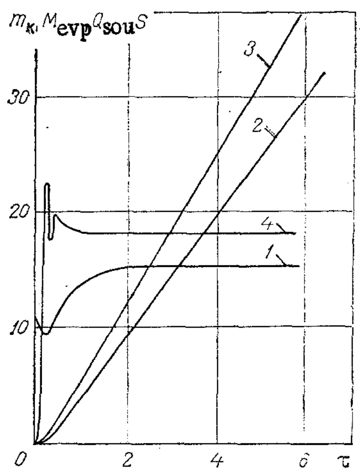

Fig. 4

Fig. 3. Distribution of pressure (1), temperature (2), density (3), of the axial velocity component (4), of the radial velocity component (5), of the magnetic field intensity (6), of the magnetic pressure (7), and of the electric conductivity of the plasma (8) at the edge of the capillary along the radius.

Fig. 4. Dependence of the plasma mass in the capillary (1), of the evaporated mass (2), of the energy liberated by the Joulean source (3), of the density of the radiant energy flux incident on the surface of the capillary (4) on the time τ, μsec.

volumetric electromagnetic force under the conditions of heavy-current discharge, non-steady-state gasdynamic processes in the capillary and outside it, and it describes both the initial, non-steady-state phase of plasma motion and the phase of steady-state flow arising as a result of the stabilization of the radiant heat fluxes at the wall and correspondingly of the mass flow of erosive plasma.

NOTATION

$Re_H = WL/D_H$, magnetic Reynolds number; $D_H = c^2/4\pi\eta$, diffusion coefficient of the magnetic field; $\mathbf{W}=\{W_r, W_\varphi, W_z\}$, speed; L, characteristic dimension; c, speed of light; η, electric conductivity of plasma; $\mathbf{H}=\{H_r, H_\varphi, H_z\}$, magnetic field intensity; $\mathbf{E}=\{E_r, E_\varphi, E_z\}$, electric field intensity; $\mathbf{j}=\{j_r, j_\varphi, j_z\}$, electric current density; ρ, density; P, pressure; ε, specific full energy (without the energy of the magnetic field taken into account; S, density of radiant energy flux; *, symbol denoting characteristic dimensional magnitudes; μ, cosine of the angle between the z axis and the direction of the light beam; φ, angle between the radius drawn to the examined point and the projection of the beam onto the plane perpendicular to the z axis; I, integral intensity of the radiation (over the spectrum); I_p, integral intensity of equilibrium radiation; κ, absorption coefficient averaged according to Planck; I^r, I^z, intensity of radiation along the r axis and z axis, respectively; $I^r_{1,2}$, $I^z_{1,2}$, mean one-sided intensities of radiation.

LITERATURE CITED

1. M. I. Demidov, N. N. Ogurtsova, and I. V. Podmoshenskii, "Pulsed light source with radiation similar to the radiation of a blackbody at 40,000°K," Opt.-Mekh. Promyshlennost', No. 1, 1-5 (1960).
2. M. I. Demidov, N. N. Ogurtsova, I. V. Podmoshenskii, and V. M. Shelemina, "Small plasma brightness standard for rapid spectroscopy," Zh. Prikl. Spektrosk., 23, No. 5, 957-960 (1975).
3. N. N. Ogurtsova, I. V. Podmoshenskii, and V. L. Smirnov, "Observation of phase transition in strongly nonideal plasma," Pis'ma Zh. Tekh. Fiz., 1, No. 22, 1049-1053 (1975); "The phenomenon of ohmic overheating in dense plasma of capillary discharge," Teplofiz. Vys. Temp., 14, No. 1, 1-9 (1976); "The phenomenon of phase transition in dense plasma of capillary discharge," Teplofiz. Vys. Temp., 15, No. 3, 456-464 (1977).

4. S. N. Belov, A. N. Zhilin, N. N. Ogurtsova, and I. V. Podmoshenskii, "Magnetogasdynamic regime of heavy-current capillary discharge," Teplofiz. Vys. Temp., 16, No. 3, 473-481 (1978).
5. V. B. Voronich, N. N. Ogurtsova, I. V. Podmoshenskii, and P. N. Rogovtsev, "Stabilization of a heavy-current pinching discharge by plasma scavenging," Zh. Tekh. Fiz., 50, No. 5, 1009-1014 (1980).
6. L. D. Landau and E. M. Lifshits, Electrodynamics of Continuous Media, Pergamon (1960).
7. O. M. Belotserkovskii and Yu. M. Davydov, "The non-steady-state method of 'large particles' for gasdynamic calculations," Zh. Vychisl. Mat. Mat. Fiz., 11, No. 1, 182-207 (1971).
8. G. S. Romanov and V. E. Okunev, Calculation of Radiative and Gasdynamic Processes Occurring in a Capillary Discharge from an Evaporating Wall [in Russian], Minsk (1979), (Report of the A. N. Sevchenko Research Institute of Applied Physical Problems, Gov. Reg. No. B880310).
9. B. Davison, The Theory of Neutron Transport [Russian translation], Atomizdat, Moscow (1960).
10. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High Temperature Hydrodynamic Phenomena, Academic Press.
11. A. A. Samarskii and Yu. P. Popov, Difference Methods of Solving Problems of Gasdynamics [in Russian], Nauka, Moscow (1980).
12. S. I. Kas'kova, G. S. Romanov, K. L. Stepanov, and V. I. Tolkach, "Coefficients of continuous absorption of carbon plasma in the temperature range up to 100 eV," Opt. Spektrosk., 46, No. 4, 655-662 (1979); G. S. Romanov, K. L. Stepanov, and M. I. Syrkin, "Spectral and mean absorption coefficients of carbon plasma," Opt. Spektrosk., 47, No. 5, 860-868 (1979).
13. N. N. Kalitkin, L. V. Kuz'mina, and V. S. Rogov, Tables of Thermodynamic Functions and of Transport Coefficients of Plasma [in Russian], (Preprint of IPM AN SSSR), Moscow (1972).

HYDRODYNAMIC ASPECT OF THE EFFECT OF THE THERMOPHYSICAL PROPERTIES OF THE HEATER ON THE CRITICAL HEAT FLUX IN BULK POOL BOILING OF HELIUM

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The article examines the hydrodynamic aspect of the effect of the thermophysical properties of the heater on the critical heat flux in pool boiling of helium.

The recently begun promising utilization of superconductivity stimulated interest in processes of heat exchange in cryogenic liquids near critical bulk boiling. Several authors [1-4] draw attention to the distinct influence of the thermophysical properties of the heater on the first critical heat flux. Sometimes the assumption is expressed that it is impossible to take this phenomenon into account within the framework of the hydrodynamic theory (e.g., [4]). We want to point out that the attempt of Grigor'ev et al. [4] to take into account the effect of the properties of the heater material on the first critical heat flux leads to the known formula of the hydrodynamic theory, and all the corrections connected with the properties of the material vanish if the exact solution is used instead of the interpolation formula (7) from the article by Grigor'ev et al. [4].

Below we present a physical model of the effect of the thermophysical properties of the heater on the maximum critical heat flux in boiling of helium, corresponding to the hydrodynamic nature of the crisis.

The hydrodynamic hypothesis explains the first critical heat flux density by infringement of the structure of the two-phase layer near the heater, and its rearrangement [5]. Here we

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